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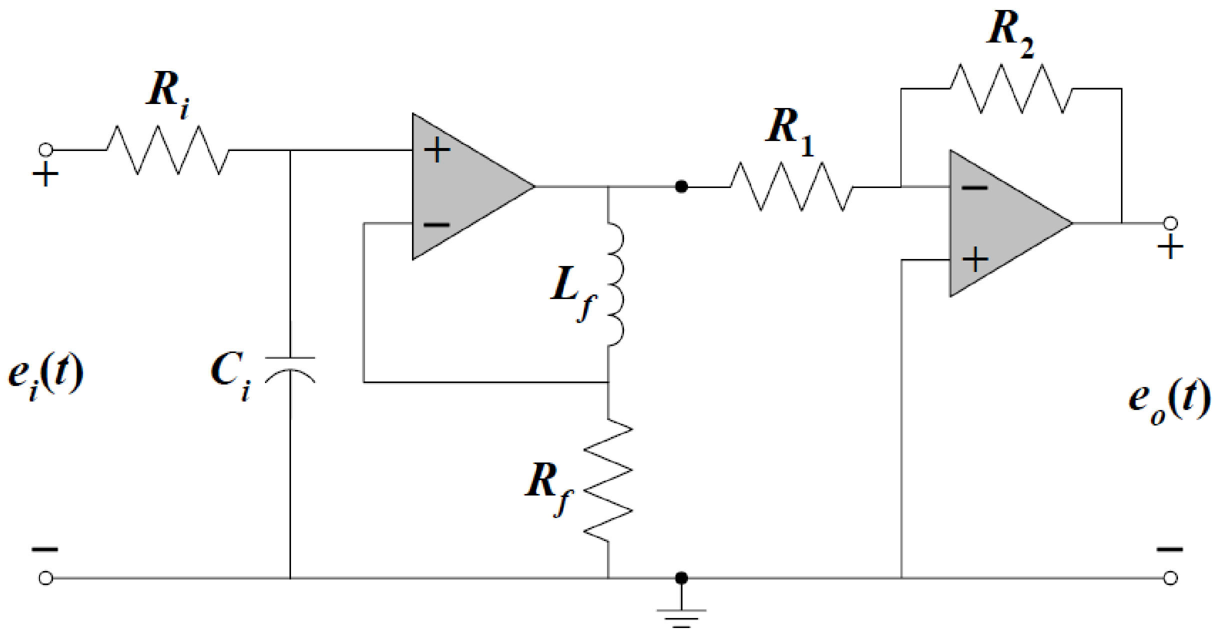

HOMEWORK #5

OPERATIONAL-AMPLIFIER SYSTEMS

ME 411: System Modeling Analysis and Control

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1.0 Nomenclature

$$Z_{Ri} = R_i = Complex\ Impedence\ of\ Input\ Resistance, R_i$$

$$Z_{ci} = \frac{1}{C_i s} = Complex\ Impedence\ of\ Input\ capacitance, C_i$$

$$Z_{Rf} = R_f = Complex\ Impedence\ of\ feebback\ resistacn, R_f$$

$$Z_{Lf} = L_f s = Complex\ Impedence\ of\ Feedback\ Inductance, L_f$$

$$Z_{R1} = R_1 = Complex\ Impedence\ of\ second\ stage\ input\ resistacne, R_1$$

$$Z_{R2} = R_2 = Complex\ Impedence\ of\ \ second\ stage\ feedback\ resistance, R_2$$

$$E_i(s) = Laplace\ transformation\ of\ input\ voltage, e_i(t)$$

$$E_0(s) = Laplace\ transformation\ of\ output\ voltage, e_0(t)$$

$$I_{Ri}(s) = Laplace\ transformation\ of\ current\ in\ Input\ Resistance, i_{Ri}(t)$$

$$I_{ci}(s) = Laplace\ transformation\ of\ current\ in\ Input\ capacitance, i_{Ci}(t)$$

$$I_{Rf}(s) = Laplace\ transformation\ of\ current\ in\ feebback\ resistacn, i_{Rf}(t)$$

$$I_{Lf}(s) = Laplace\ transformationLaplace\ transformation\ of\ current\ in\ Feedback\ Inductance, i_{lf}(t)$$

$$I_{R1}(s) = Laplace\ transformation\ of\ current\ in\ second\ stage\ input\ resistacne, i_{R1}(t)$$

$$I_{R2}(s) = Complex\ Impedence\ of\ current\ in\ second\ stage\ feedback\ resistance, i_{R2}(t)$$

$$E_A(s) = Laplace\ transformation\ of\ node\ A\ voltage, e_A(t)$$

$$E_B(s) = Laplace\ transformation\ of\ node\ B\ voltage, e_B(t)$$

$$E_B(s) = Laplace\ transformation\ of\ node\ C\ voltage, e_C(t)$$

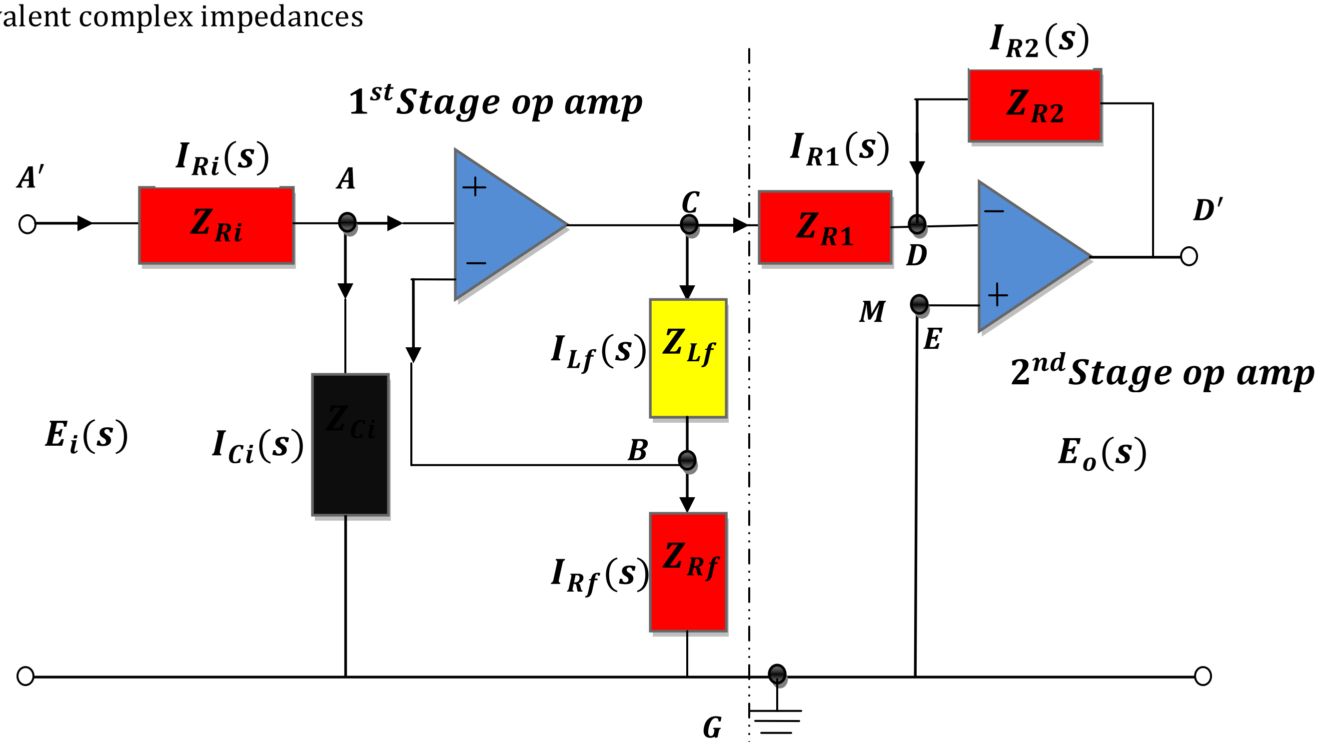
$$E_D(s) = Laplace\ transformation\ of\ node\ D\ voltage, e_D(t)$$

$$E_E(s) = Laplace\ transformation\ of\ node\ E\ voltage, e_E(t)$$

2.0 Transfer functions & governing equations.

Assume the op-amps are ideal, use the complex impedance method to find the transfer function: $\frac{E_0(s)}{E_i(s)}$, where $E_i(s)$ and $E_0(s)$ are the Laplace transforms of $e_i(t)$ and $e_0(t)$, respectively. Recover also the differential equation of the transfer function.

The RLC operational amplifier circuits substituting all the electric elements by their equivalent complex impedances



Simplifying the circuits:

Applying KVL to A' AG

$$\frac{E_i(s)-E_A(s)}{Z_{Ri}} = I_{Ri}(s) \quad \rightarrow E_i(s) = E_A(s) + (Z_{Ri})I_{Ri}(s) \qquad (1)$$

From ohm's law for impedance we get,

$$\frac{E_A(s)-0}{Z_{Ci}} = I_{Ci}(s) \quad \rightarrow E_A(s) = (Z_{Ci})I_{Ci}(s) \qquad (2)$$

Applying KVL to CBG

$$\frac{E_C(s)-E_B(s)}{Z_{Lf}} = I_{Lf}(s) \quad \rightarrow E_C(s) = E_B(s) + (Z_{Lf})I_{Lf}(s) \qquad (3)$$

From ohm's law for impedance we get,

$$\frac{E_B(s)-0}{Z_{Rf}} = I_{Rf}(s) \quad \rightarrow E_B(s) = (Z_{Rf})I_{Rf}(s) \qquad (4)$$

Since $\mathbf{e_C(t) = K[\, e_A(t) - e_B(t)]}$

For Ideal Op − amp $K \to \infty$

$$\mathbf{0 = \infty[\, e_A(t) - e_B(t)] \to e_A(t) = e_B(t)}$$

$$\boldsymbol{E_A(s) = E_B(s)} \qquad \textbf{(5)}$$

Again, $\boldsymbol{I_{op-amp}(s) = \dfrac{E_{op-amp}(s)}{Z_{op-amp}(s)}}$

For Ideal Op − amp Infinite input impedence $Z_{op-amp}(s) \to \infty$

$$\boldsymbol{I_{op-amp}(s) = \frac{E_{op-amp}(s)}{\infty} \to I_{op-amp}(s) = 0}$$

Applying KCL at nodes A and B we get,

Node A: $\boldsymbol{I_{Ri}(s) = I_{Ci}(s)}$ **(6)**

Node B: $\boldsymbol{I_{Lf}(s) = I_{Rf}(s)}$ **(7)**

Using the voltage division rule,

$$\boldsymbol{\frac{E_i(s)}{Z_{Ci} + Z_{Ri}(s)} = \frac{E_A(s)}{Z_{Ci}} \quad \to E_A(s) = \frac{Z_{Ci}}{Z_{Ci} + Z_{Ri}} E_i(s)}$$

Substituting : $\boldsymbol{Z_{Ci} = \frac{1}{C_i s}}$, $\boldsymbol{Z_{Ri} = R_i}$

$$\boldsymbol{E_A(s) = \frac{1}{1 + R_i C_i s} E_i(s)}$$

Again by voltage division rule,

$$\boldsymbol{\frac{E_C(s)}{Z_{Lf} + Z_{Rf}} = \frac{E_B(s)}{Z_{Rf}} \quad \to E_B(s) = \frac{Z_{Rf}}{Z_{Lf} + Z_{Rf}} E_C(s)}$$

Substituting: $\boldsymbol{Z_{Lf} = L_f}$, $\boldsymbol{Z_{Rf} = R_f}$

Therefore we get,

$$\boldsymbol{E_B(s) = \frac{R_f}{L_f s + R_f} E_C(s)} \qquad (9)$$

Hence,

$$\boldsymbol{E_B(s) = E_A(s)}$$

$$\boldsymbol{\frac{1}{1+R_iC_is}E_i(s) = \frac{R_f}{L_fs+R_f}E_C(s)}$$

So, the transfer function of the 1st –stage op-amp is,

$$\boldsymbol{T_1(s) = \frac{E_C(s)}{E_i(s)} = \frac{L_fs+R_f}{R_f(1+R_iC_is)}} \qquad \textbf{(10)}$$

Now, for the transfer function of 2nd – stage op- amp

Simplifying the circuits:

Applying KVL to CDG

$$\boldsymbol{\frac{E_c - E_D}{Z_{R1}} = I_{R1}(s) \;\; \rightarrow \;\; E_c(s) = \; E_D(s) + Z_{R1}I_{R1}(s)} \qquad \textbf{(11)}$$

Applying KVL to D'DG

$$\boldsymbol{\frac{E_0 - E_D}{Z_{R2}} = I_{R2}(s) \;\; \rightarrow \;\; E_0(s) = \; E_D(s) + Z_{R2}I_{R2}(s)} \qquad \textbf{(12)}$$

Since $e_O(t) = Ke_{op-amp}(t) = K[\, e_E(t) - e_D(t)]$ is finite and the 2nd –stage op-amp is ideal, therefore,

$$\text{For Ideal Op} - \text{amp } K \rightarrow \infty$$

$$\mathbf{0 = \infty[\, \cancel{e_E(t)} - e_D(t)] \rightarrow e_D(t) = 0}$$

$$\boldsymbol{E_D(s) = 0} \qquad \textbf{(13)}$$

$$\text{For Ideal Op} - \text{amp Infinite input impedence } Z_{op-amp}(s) \rightarrow \infty$$

$$\boldsymbol{I_{op-amp}(s) = \frac{E_{op-amp}(s)}{\infty} \rightarrow I_{op-amp}(s) = 0}$$

Applying KCL at nodes A and B we get,

$$\boldsymbol{\textit{Node D: } I_{R1}(s) = -I_{R1}(s)} \qquad \textbf{(14)}$$

Substituting $Z_{R1}(s) = R_1 \; and \; Z_{R2}(s) = R_2$ and we get,

$$\boldsymbol{E_c(s) = \cancel{E_D(s)} + R_1\, I_{R1}(s) \;\; and \; E_0(s) = \cancel{E_D(s)} + R_2I_{R2}(s)}$$

Hence,

$$\boldsymbol{I_{R1}(s) = -I_{R2}(s)}$$

$$\boldsymbol{\frac{E_c(s)}{R_1} = -\frac{E_0(s)}{R_2}}$$

$$\boldsymbol{\frac{E_0(s)}{E_c(s)} = -\frac{R_2}{R_1}}$$

Therefore, the entire transfer function for the op-amp circuit we get,

$$\boldsymbol{T(s) = \frac{E_o(s)}{E_i(s)}}$$

$$\boldsymbol{T(s) = \frac{-\frac{R_2}{R_1}\cancel{E_c(s)}}{\frac{L_f+R_f}{R_f(1+R_iC_is)}\cancel{E_c(s)}} = -\frac{R_2}{R_1}\frac{L_f+R_f}{R_f(1+R_iC_is)}}$$

$$\boldsymbol{T(s) = \frac{E_o(s)}{E_i(s)} = -\frac{R_2(L_fs+R_f)}{R_1R_f(1+R_iC_is)}}$$

$$\boldsymbol{R_1R_f(1+R_iC_is) * E_o(s) = -E_i(s) * R_2(L_fs+R_f)}$$

The differential equation of the transfer function is:

$$\boldsymbol{e_0(t) + R_iC_i\frac{de_0(t)}{dt} = -\frac{R_2}{R_1}e_i(t) - \frac{R_2L_f}{R_1R_f}\frac{de_i(t)}{dt}}$$

3.0 System parameters

$$\boldsymbol{e_0(t) + R_iC_i\frac{de_0(t)}{dt} = -\frac{R_2}{R_1}e_i(t) - \frac{R_2L_f}{R_1R_f}\frac{de_i(t)}{dt}}$$

The system is first order.

$$\tau\frac{de_0(t)}{dt} + e_0(t) = \sum_{j=0}^{m}\left[K_j\frac{d^je_i(t)}{dt^j}\right]$$

Time constant $(\tau) = \boldsymbol{R_iC_i}$

The static sensitivities $K_0 = -\boldsymbol{\frac{R_2}{R_1}}, K_1 = -\boldsymbol{\frac{R_2L_f}{R_1R_f}}$

The zeros and poles of the transfer function (S_{pole}) = $\boldsymbol{-\frac{1}{C_i R_i}}$, ($S_{zero}$) = $\boldsymbol{-\frac{R_f}{L_f}}$

4.0 The 2nd –stage Op-amp:

Since for the second stage op amp is an inverting operational amplifier, the purpose of asserting this component is to reversed the polarity of the output from the first stage operational amplifier and adjust the gain as output is multiplied by the ratio of input resistance and feedback resistance. The ratio of these second stage resistors also governs the overall gain of the circuit.

5.0 Steady-State Response

If the input voltage , $e_i(t)$ is a step function, i.e.

$$e_i(t) = \overline{e_\iota} * 1(t) = \begin{cases} \overline{e_\iota} & for\ t > 0 \\ 0 & for\ t < 0 \end{cases}$$

where the constant $\overline{e_\iota}$ is the step input voltage, determine the steady-state output voltage $e_{0ss} = \lim_{t\to\infty} e_0(t)$. . If possible, estimate the time required to reach this steady state. If not, justify your reason why not?

Using the final value theorem,

$$\frac{E_o(s)}{E_i(s)} = -\frac{R_2(L_f s + R_f)}{R_1 R_f(1 + R_i C_i s)}$$

$$E_0(s) = -\frac{R_2(L_f s + R_f)}{R_1 R_f(1 + R_i C_i s)} E_i(s)$$

$$For, e_{0ss} = \lim_{t\to 0} e_0(t) = \lim_{s\to 0} sE_0(s)$$

$$e_{0ss} = \lim_{s\to 0} \cancel{s}\left(-\frac{R_2(L_f s + R_f)}{R_1 R_f(1 + R_i C_i s)}\right)\frac{\overline{e_\iota}}{\cancel{s}}$$

$$e_{0ss} = \left(-\frac{R_2}{R_1}\right)\overline{e_\iota}$$

This physically implies for step input voltage as time tends to infinity the capacitor (C_i) behaves as short circuit and under full saturation it doesn't contribute to the circuit output. However the ratio of second stage op amp feedback resistor and input resistor governs the final steady state value, for simplification if R_1 equals to R_2 the steady state value of the output equals input value. The output is also is in the reversed polarity of that of input.

The time required to reach the steady state is 1% of the settling time (t_s 99%)=5τ=$5\ R_i C_i$